Answer ALL TWENTY SIX questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Expand and simplify $(y + 4)(2 - y)$

$y \times 2 = 2y$

$y \times -y = -y^2$

$4 \times 2 = 8$

$4 \times -y = -4y$

$-y^2 - 2y + 8$

(2)

(b) Factorise fully $15b^5c - 35b^3c^9$

$5b^3c(3b^2 - 7c^8)$

(2)

(Total for Question 1 is 4 marks)

2 Show that $6\frac{3}{4} \div 2\frac{4}{7} = 2\frac{5}{8}$

$6\frac{3}{4} = \frac{27}{4}$

$2\frac{4}{7} = \frac{18}{7}$

$\frac{27}{4} \div \frac{18}{7}$

$= \frac{\cancel{27}^{3}}{4} \times \frac{7}{\cancel{18}_{2}}$

$= \frac{21}{8}$

$\frac{21}{8} = 2\frac{5}{8}$ as required

(Total for Question 2 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

3

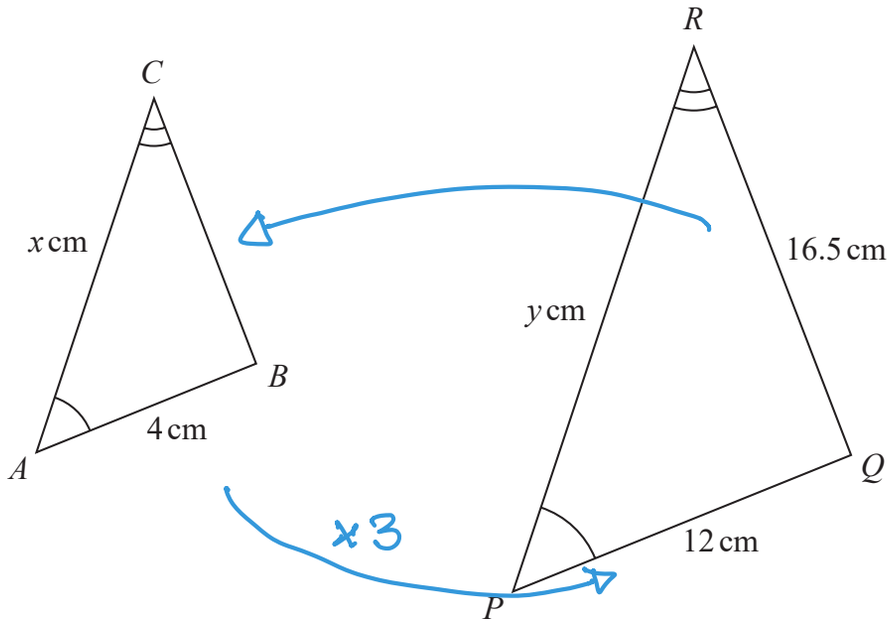


Diagram **NOT** accurately drawn

Triangle ABC is similar to triangle PQR

$AB = 4$ cm $PQ = 12$ cm $RQ = 16.5$ cm $AC = x$ cm $PR = y$ cm

(a) Calculate the length of BC

16·5 ÷ 3 =

5·5 cm

(2)

(b) Write down an expression for y in terms of x

y = 3x

$y =$ 3x

(1)

(Total for Question 3 is 3 marks)

4 Each side of a regular octagon has a length of 18 mm, correct to the nearest 0.5 mm

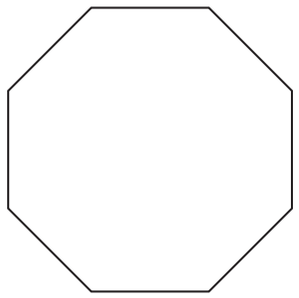


Diagram **NOT** accurately drawn

(a) Write down the lower bound of the length of each side of the octagon.

17.5 ↑ 18 ↑ 18.5

17.75 ..... mm

(1)

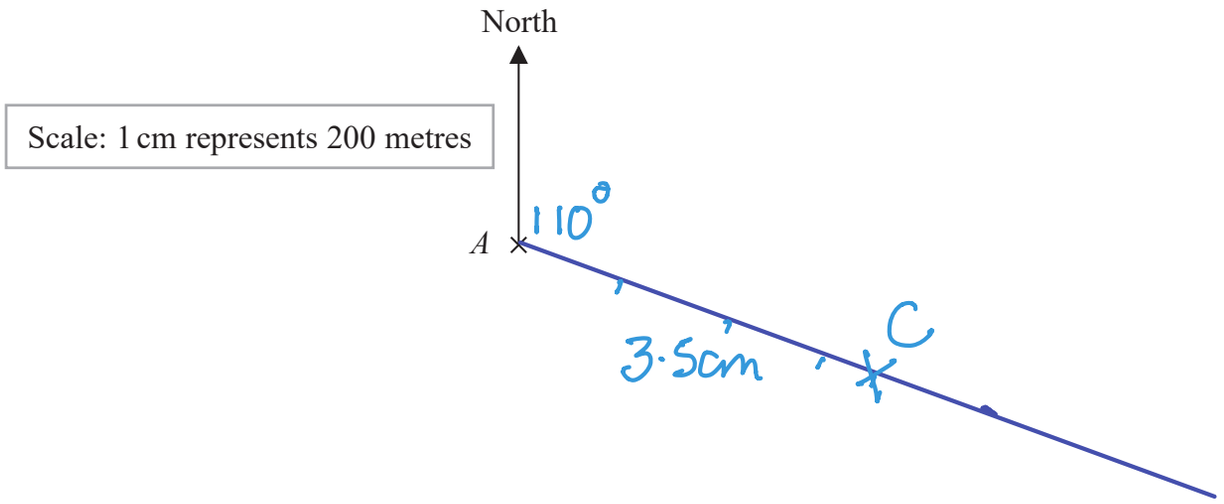
(b) Write down the upper bound of the length of each side of the octagon.

18.25 ..... mm

(1)

(Total for Question 4 is 2 marks)

5 The scale diagram shows the position on a map of a house, A



House C is on a bearing of 110° from A
The distance from A to C is 700 m

(a) Mark the position of C on the diagram with a cross (×)
Label your cross C

1cm = 200m
$\frac{1}{2}$cm = 100m
3cm = 600m
3·5cm = 700m

(3)

(b) Write the scale of the map in the form $1 : n$

1cm = 200m
1cm = 20000

1 : 20000

(1)

(Total for Question 5 is 4 marks)

6 A bag contains only pink sweets, white sweets, green sweets and red sweets.

The table gives each of the probabilities that, when a sweet is taken at random from the bag, the sweet will be green or the sweet will be red.

Sweet	pink	white	green	red
Probability	0·3	0·15	0.2	0.35

The ratio
number of pink sweets : number of white sweets = 2 : 1

There are 28 red sweets in the bag.

Work out the number of white sweets in the bag.

$1 - (0.2 + 0.35) = 0.45$

P : W
2 : 1

$0.45 \div 3 = 0.15$

0·3 : 0·15

RED

$0.35 = 28$

÷7 ↓ $0.05 = 4$ sweets

×3 ↓ $0.15 = 12$ sweets

12

(Total for Question 6 is 5 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

7 Find the lowest common multiple (LCM) of 28, 42 and 63
Show your working clearly.

28	42	63
x	✓	126
x	x	189
✓	✓	252

252

(Total for Question 7 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

8 The table gives information about the average house price in England in 2018 and in 2019

Year	2017	2018	2019
Average house price (£)		228 314	231 776

(a) Work out the percentage increase in the average house price from 2018 to 2019
Give your answer correct to one decimal place.

231776 - 228318 = 3462

3462 / 228314 x 100 = 1.5163...

1 dp

1.5 %

(2)

The average house price in 2019 was 7.7% greater than the average house price in 2017

(b) Work out the average house price in 2017
Give your answer correct to 3 significant figures.

107.7 % = 231776

÷ 107.7

1 % = 2152.05 ...

x 100

100 % = 215205.1996

3sf.

£ 215000

(3)

(Total for Question 8 is 5 marks)

9 The frequency table gives information about the number of points scored by a player.

Number of points	Frequency
0	13
1	17
2	8
3	x
4	11

$4 + x$

The mean number of points scored is 2

Work out the value of x

$0 \times 13 + 1 \times 17 + 2 \times 8 + 3 \times x + 4 \times 11 =$

$0 + 17 + 16 + 3x + 44 =$

$77 + 3x$

Mean $\quad 2 = \dfrac{77 + 3x}{4 + x}$

$98 + 2x = 77 + 3x$

$98 - 77 = 3x - 2x$

$21 = x$

$x =$ 21

(Total for Question 9 is 4 marks)

10 Solve the simultaneous equations

$$3x + 5y = 3.1$$
$$6x + 3y = 3.75$$

Show clear algebraic working.

$3x + 5y = 3.1$ (1)

$6x + 3y = 3.75$ x 2 (2)

$6x + 10y = 6.2$ (3)

$6x + 3y = 3.75$ (2)

(3) - (2)

$7y = 2.45$

$y = 0.35$

sub into (2)

$6x + 3 \times 0.35 = 3.75$

$6x = 3.75 - 1.05$

$6x = 2.7$

$x = \frac{2.7}{6}$

$x = 0.45$

$x =$ 0.45

$y =$ 0.35

(Total for Question 10 is 3 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

11 The diagram shows a regular 10-sided polygon, *ABCDEFGHIJ*

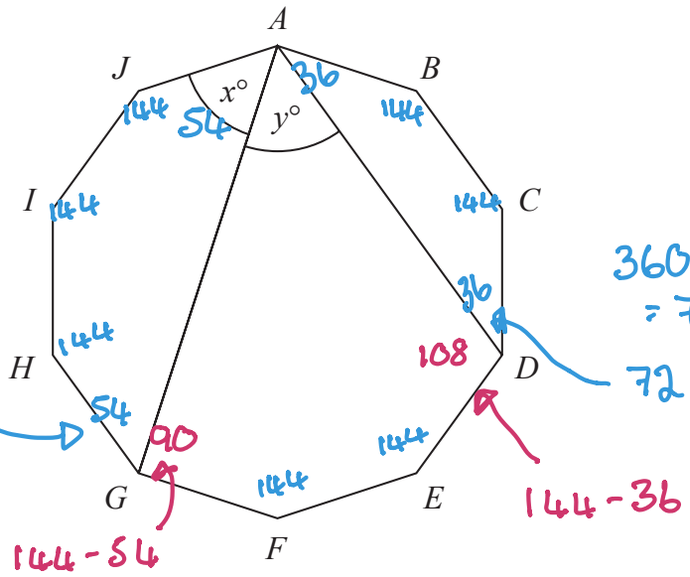


Diagram **NOT** accurately drawn

540 - (144 x 3)
= 108
108 ÷ 2 = 54

360 - (144 x 2)
= 72
72 ÷ 2 = 36

Show that $x = y$

exterior angle = $\frac{360}{10}$ = 36

so interior angle = 180 - 36 = 144

x = 54

y 540 - (90 + 108 + 144 + 144)

= 540 - 486

= 54

$\therefore x = y = 54$

(Total for Question 11 is 4 marks)

12 $a = 6 \times 10^{40}$

Work out the value of a^3
Give your answer in standard form.

$(6 \times 10^{40})^3$

$= 6^3 \times 10^{40 \times 3}$

$= 216 \times 10^{120}$

$= 2.16 \times 10^{122}$

2.16×10^{122}

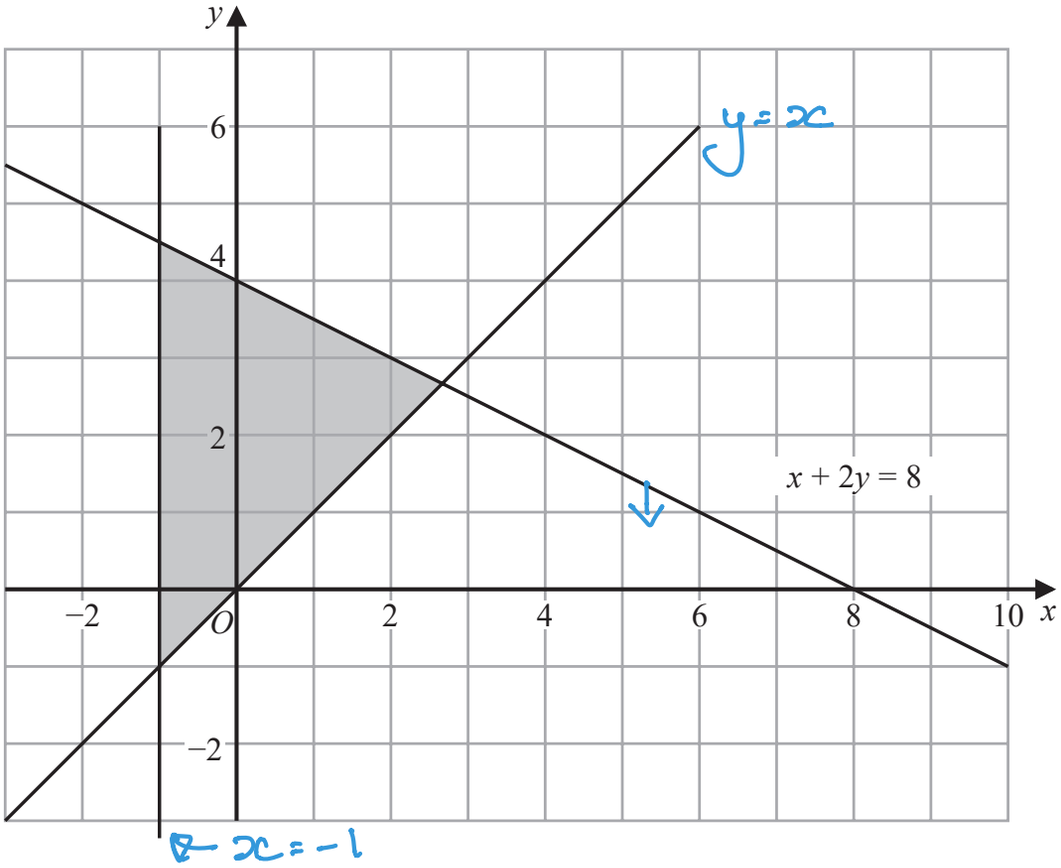
(Total for Question 12 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

13 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down three inequalities that define the shaded region.

$y \geqslant x$

$x \geqslant -1$

$x \geqslant -1$

$y \geqslant x$

$x + 2y \leqslant 8$

(Total for Question 13 is 3 marks)

14 A zip wire is shown as the dashed line AC in the diagram.

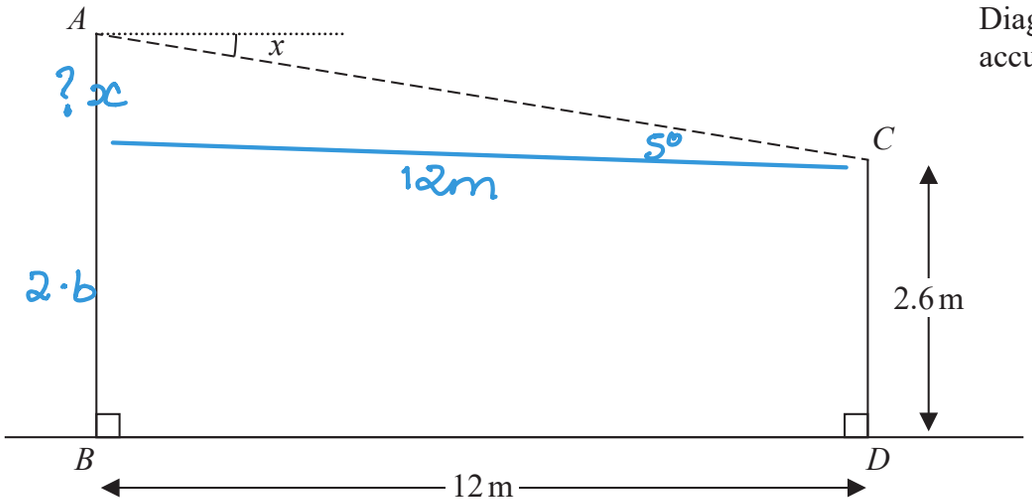


Diagram **NOT** accurately drawn

The zip wire is supported by two vertical posts AB and CD standing on horizontal ground.

$CD = 2.6\,\text{m}$ $BD = 12\,\text{m}$

The zip wire makes an angle x with the horizontal, as shown in the diagram.
The design of the zip wire requires the angle x to be at least 5°

Work out the least possible height of the post AB
Give your answer correct to 3 significant figures.

$\tan 5 = \frac{x}{12}$

$x = 12 \tan 5$

$= 1.049...$

min height $= 2.6 + 1.049...$

$= 3.6498...$

3.65 m

(Total for Question 14 is 3 marks)

15 Diyar recorded the distance, in kilometres, that he cycled each day for 11 days.
Here are his results.

8 10 12 13 5 23 21 7 5 16 14

5 5 7 8 10 12 13 14 16 21 23

Find the interquartile range of his results.

median = 12

LQ = 7

UQ = 16

IQR = 16 − 7 = 9

9 km

(Total for Question 15 is 3 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

16 D, E, F and G are points on a circle, centre O

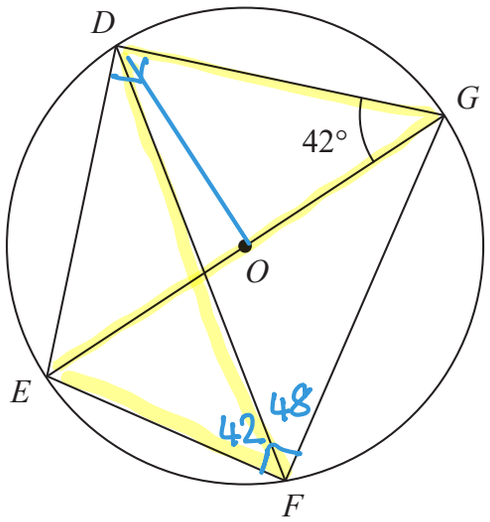


Diagram **NOT** accurately drawn

EOG is a diameter of the circle.

Angle $EGD = 42°$

Calculate the size of angle DFG
Give a reason for each stage of your working.

$\hat{DFE} = \hat{DGE} = 42$ angles in the same segment are equal

$\triangle EDC$ and EGF angles ina semicircle = 90°

$\hat{EDG} = EFG = 90$

$90 - 42 = 48$

Angle DFG = 48 °

(Total for Question 16 is 4 marks)

17 Show that $\dfrac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a + \sqrt{b}$ where a and b are integers.

$\sqrt{12} = \sqrt{3}\sqrt{4} = 2\sqrt{3}$

$\dfrac{2\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{4\sqrt{3} - 2\sqrt{3}\sqrt{3}}{4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{3}\sqrt{3}} = \dfrac{4\sqrt{3} - 6}{4 - 3}$

$= \dfrac{4\sqrt{3} - 6}{1} = -6 + 4\sqrt{3}$

form $a + \sqrt{b} \Rightarrow -6 + \sqrt{16}\sqrt{3}$

$= -6 + \sqrt{48}$

$a = -6$

$b = 48$

(Total for Question 17 is 3 marks)

18 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

let any numbe = n so any even numbe is 2n

any odd numbe = 2n + 1

consecutive odd numbe = 2n + 3

SUM OF SQUARES

$(2n+1)^2 + (2n+3)^2$

$= 4n^2 + 4n + 1 + 4n^2 + 12n + 9$

$= 8n^2 + 16n + 10$

DIVIDED BY 8 $\quad \frac{8n^2}{8} + \frac{16n}{8} + \frac{10}{8}$

$= n^2 + 2n + \frac{10}{8}$

$10 - 8 = 2$

(Total for Question 18 is 3 marks)

19

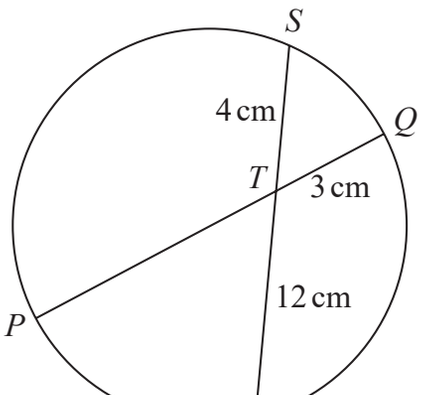


Diagram **NOT** accurately drawn

PTQ is a diameter of a circle.
RTS is a chord of the circle.

$TQ = 3\,\text{cm}$ $ST = 4\,\text{cm}$ $TR = 12\,\text{cm}$

Calculate the radius of the circle.

$PT = \frac{12 \times 4}{3} = 16$

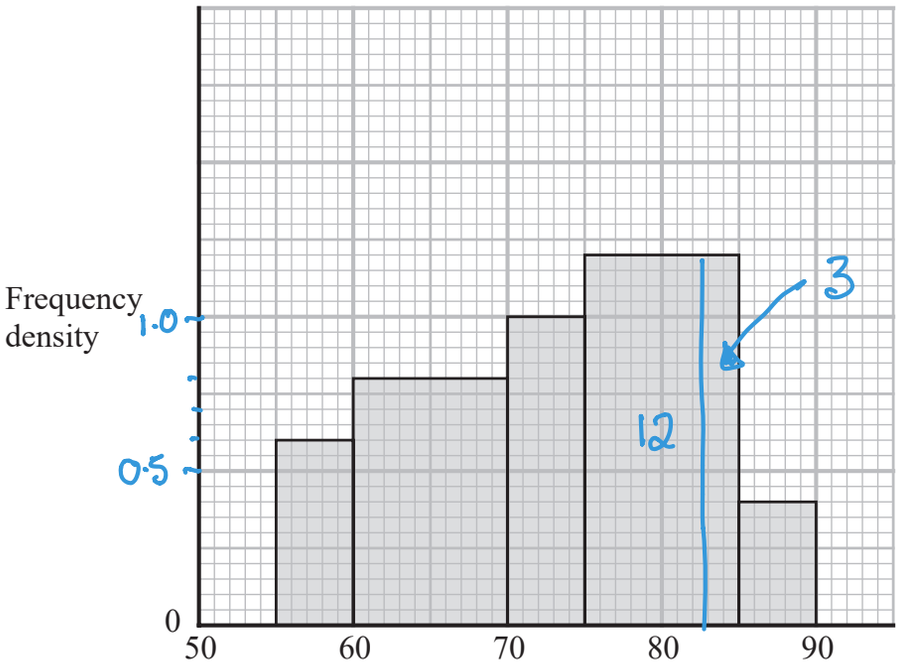
Diameter $= 16 + 3 = 19$

so radius $= 9.5$

9.5 cm

(Total for Question 19 is 3 marks)

20 The histogram gives information about the heights, h cm, of some tomato plants.



There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

F.D. = 12 ÷ 10 = 1·2

50 - 60 0·6 x 5 = 3

60 - 70 0·8 x 10 = 8

70 - 75 1 x 5 = 5

75 - 85 = 12

85 - 90 0·4 x 5 = 2

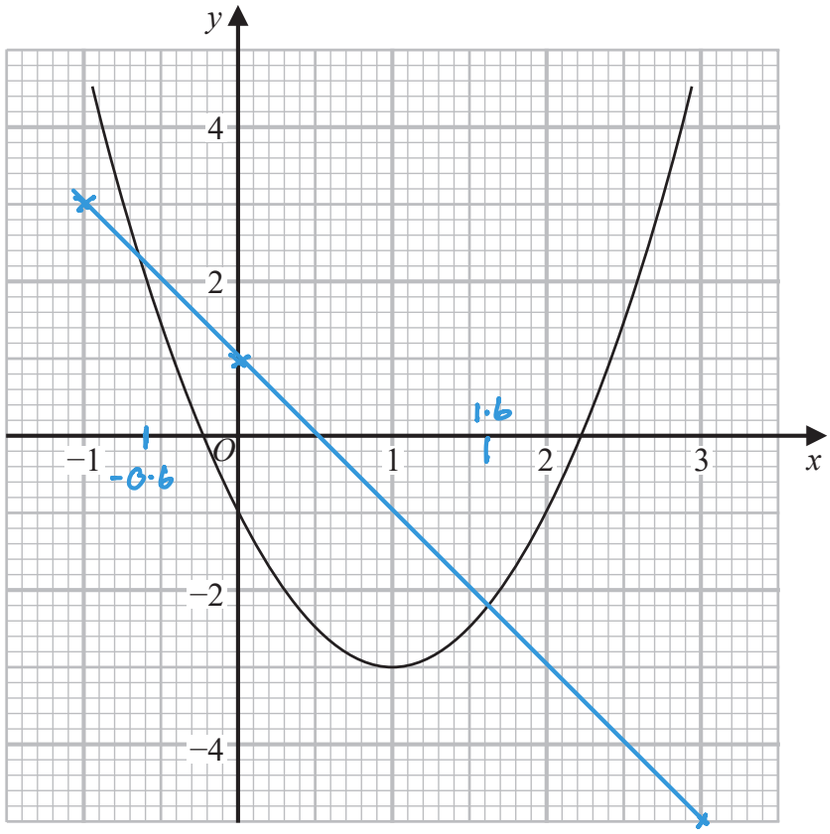
30

> 82.5 = 3 + 2 = 5

$\frac{5}{30} = \frac{1}{6}$

(Total for Question 20 is 4 marks)

21 Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.



(a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.

-0.2 , 2.2

(2)

(b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
Show your working clearly.
Give your solutions correct to one decimal place.

$y = 2x^2 - 4x - 1$

$x^2 - x - 1 = 0$

$2x^2 - 2x - 2$

$y = -2x + 1$

$x = -1 \quad y = 3$
$x = 0 \quad y = 1$
$x = 3 \quad y = -5$

$x = -0.6 \quad x = 1.6$

(3)

(Total for Question 21 is 5 marks)

22 Here is a rectangle.

Diagram **NOT** accurately drawn

Given that the area of the rectangle is less than 75 cm^2

find the range of possible values of x

$(2x+3)(x-1) < 75$

$2x^2 - 2x + 3x - 3 - 75 < 0$

$2x^2 + x - 78 < 0$

$(x-6)(2x+13) < 0$

$x = 6$ $\quad$ $x = -6.5$ not valid $\quad$ $x < 6$

$2x + 3 \Rightarrow 1$

$x - 1 \Rightarrow x$ cannot be < 1

$1 < x < 6$

(Total for Question 22 is 5 marks)

23 The diagram shows triangle PQR

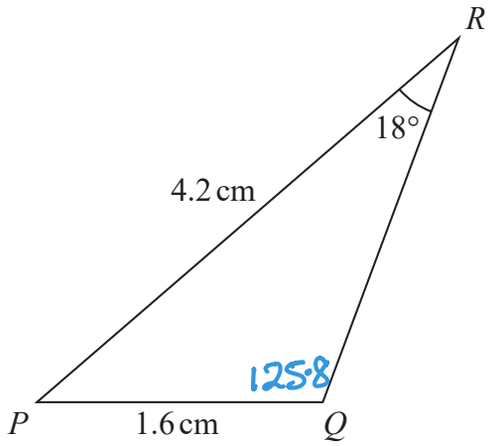


Diagram **NOT** accurately drawn

$PQ = 1.6$ cm $PR = 4.2$ cm Angle $PRQ = 18°$

Given that angle PQR is obtuse, ← LOOK !!

work out the area of triangle PQR
Give your answer correct to 3 significant figures.

$\frac{\sin Q}{4.2} = \frac{\sin 18}{1.6}$

$\sin Q = \frac{\sin 18}{1.6} \times 4.2$

$= 54.210...$ but PQR is obtuse so PQR = 125.8

$RPQ = 180 - (18 + 125.8..)$

$= 36.2$

$\text{Area} = \frac{1}{2}\ 4.2 \times 1.6 \times \sin 36.2$

$= 1.984..$

1.98 ..... cm^2

(Total for Question 23 is 6 marks)

24 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a\ \text{m/s}^2$

Find the value of a

$v = 12t^2 - 27$

$12t^2 - 27 = 0$

$12t^2 = 27$

$t^2 = \frac{27}{12}$ $\quad t = \sqrt{\frac{27}{12}} = 1.5$

$a = 24t$

when $t = 1.5$

$= 24 \times 1.5$

$a =$ 36

(Total for Question 24 is 5 marks)

25 The function g is defined as

$$g: x \mapsto 5 + 6x - x^2 \quad \text{with domain } \{x : x \geqslant 3\}$$

(a) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \ldots$

$y = 5 + 6x - x^2$

$= 5 - [(x-3)^2 - 9]$

$y = 14 - (x-3)^2$

$(y - 14) = -(x-3)^2$

$14 - y = (x-3)^2$

$x = 3 \pm \sqrt{14-y}$

$g^{-1} \rightarrow 3 + \sqrt{14-x}$

given $x \geqslant 3$

$g^{-1}: x \mapsto$ $3 + \sqrt{14-x}$

(4)

(b) State the domain of g^{-1}

$x \leqslant 14$

(1)

(Total for Question 25 is 5 marks)

26 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$S_m = \frac{m}{2}(2a + (m-1)d) = 39$ $\quad$ $S_{2m} = \frac{2m}{2}(2a + (2m-1)d) = 320$

$2ma + (m-1)md = 78$ $\quad$ ② $2ma + 2m^2d - md = 320$

$2ma + m^2d - md = 78$ ①

$$\begin{array}{rrl} & 2ma + 2m^2d - md = 320 & \\ - & 2ma + m^2d - md = 78 & \\ \hline & m^2d & x = 242 \end{array}$$

d is a prime number... so $242 = 2 \times 11^2$

$\therefore d = 2 \quad m = 11$

$d =$ 2

$m =$ 11

(Total for Question 26 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS